

MATHEMATICS COMMON CORE CURRICULUM UNIT #2 Algebra 2*

North Smithfield School Department

TITLE OF UNIT: Unit 2 Rational and Radical Functions **COURSE:** Algebra 2

DATE PRESENTED: _____ **DATE DUE:** _____ **LENGTH OF TIME:** Several weeks, quarter, semester

OVERVIEW OF UNIT:

Unit 2 standards will focus on rational and radical functions. Topics include performing operations with rational expressions, solving equations involving rational and radical functions, and graphing simple rational and radical functions. Solving equations by graphing is also explored.

**ESSENTIAL QUESTION,
PROMPT, PROBLEM/UNIT**

STANDARDS: Common Core Math Standards – Grade level Categories 9-12

Number and Quantity	Algebra	Functions	Modeling	Geometry	Statistics and Probability
<input type="checkbox"/> The Real Number System N-RN	<input type="checkbox"/> Seeing Structure in Expressions A-SSE	<input type="checkbox"/> Interpreting Function F-If	<input type="checkbox"/>	<input type="checkbox"/> Congruence G-CO	<input type="checkbox"/> Interpreting Categorical and Quantitative Data S-ID
<input type="checkbox"/> Quantities N-Q	<input type="checkbox"/> Arithmetic with Polynomials and Rational Expressions A-APR	<input type="checkbox"/> Building Functions F-BF	<input type="checkbox"/>	<input type="checkbox"/> Similarity, Right Triangles, and Trigonometry G-SRT	<input type="checkbox"/> Making Inferences and Justifying Conclusions S-IC
<input type="checkbox"/> The Complex Number System N-CN	<input type="checkbox"/> Creating Equations A-CED	<input type="checkbox"/> Linear, Quadratic, and Exponential Models F-LE	<input type="checkbox"/>	<input type="checkbox"/> Circles G-c	<input type="checkbox"/> Using Probability to Make Decisions S-MD
<input type="checkbox"/> Vector and Matrix Quantities N-VM	<input type="checkbox"/> Reasoning with Equations and Inequalities A-REI	<input type="checkbox"/> Trigonometric Functions F-TF	<input type="checkbox"/>	<input type="checkbox"/> Expressing Geometric Properties with Equations G-GPE	
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> Geometric Measurement and Dimensions G-GMD	
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> Modeling with Geometry G-MG	

STANDARDS: Mathematical Practices grades K-12

- | | | | | |
|---|--|--|---------------------------------------|--|
| 1. Make sense of problems and persevere in solving them | 3. Construct viable arguments and critique the reasoning of others | 5. Use appropriate tools strategically | 7. Look for and make use of structure | 8. Look for and express regularity in repeated reasoning |
| 2. Reason abstractly and quantitatively | 4. Model with mathematics ★ | 6. Attend to precision | | |

FOCUS MATHEMATICS STANDARDS:

- Interpret the structure of expressions. A-SSE.1a,b
- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. A-REI.1
- Rewrite rational expressions. A-APR.6, (+)7
- Represent and solve equations and inequalities graphically. A-REI.11
- Analyze functions using different representations. F-IF.7b, 7d(+), 8a, 9

Applied Learning Standards:

problem solving communication critical thinking research reflection/ evaluation

Expectations for Student Learning (High School only):

Problem Solving, Communication, Body of Knowledge, Responsibility

ENDURING UNDERSTANDING:

At the end of this unit, students will be proficient in the following:

- Perform operations with rational expressions.
- Solve equations involving rational and radical functions, including extraneous solutions.
- Graph simple rational and radical functions by hand, and more complex functions using technology.
- Rewrite rational expressions.
- Represent and solve equations and inequalities graphically.
- Analyze functions using different representations.

MATHEMATICS COMMON CORE CURRICULUM UNIT #2 Algebra 2*

North Smithfield School Department

PRIOR KNOWLEDGE:

Algebra 1 and Geometry

STUDENT OBJECTIVES, SKILLS and/or NEW KNOWLEDGE:

- A rational expression is a quotient of two polynomials; the denominator must be nonzero.
- Rational expressions can be written in different forms using factoring and arithmetic operations.
- An expression of the form $\frac{a(x)}{b(x)}$ can be written as $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials, and the degree of $r(x)$ is less than the degree of $b(x)$.
- Inspection and long division are two methods for rewriting a rational expression.
- (+)Adding, subtracting, multiplying, and dividing rational expressions result in another rational expression, thus making it a closed system.
- (+)Adding, subtracting, multiplying, and dividing rational expressions follow the same rules as operations on rational numbers.
- Simple rational and radical equations can have extraneous solutions.
- Solving a system of equations algebraically yields an exact solution; solving by graphing or by comparing tables of values yields an approximate solution.
- The x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.
- Key features of a graph or table may include intercepts; intervals in which the function is increasing, decreasing or constant; intervals in which the function is positive, negative or zero; symmetry; maxima; minima; end behavior; asymptotes; domain; range and periodicity.
- Graph square root and cube root functions.
- (+)Graph rational functions.
- A function can be represented algebraically, graphically, numerically in tables, or by verbal descriptions.

SUGGESTED PROBLEMS:

Teaching Examples A-SSE.1

In Algebra I, students work with linear, exponential, and quadratic expressions. In Algebra II, students extend these concepts to general polynomials and rational expressions.

- Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.

Teaching Examples A-APR.6

- The limitations on rational functions apply to the rational expressions in A-APR.6.
- The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes.

Examples:

- Find the quotient and remainder for the rational expression $\frac{x^3 - 3x^2 + x - 6}{x^2 + 2}$ and use them to write the expression in a different form.

- Express $f(x) = \frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph.

- Solution: $f(x) = \frac{2x+1}{x-1} = \frac{2(x-1)+3}{x-1} = \frac{2(x-1)}{x-1} + \frac{3}{x-1} = 2 + \frac{3}{x-1}$, so the horizontal asymptote is $y = 2$. (TUSD)

(+)Teaching Examples A-APR.7

A-APR.7 requires the general division algorithm for polynomials.

Examples:

- Use your knowledge about the sum of two fractions to explain why the sum of two rational expressions is another rational expression.

- Express $\frac{1}{x^2+1} - \frac{1}{x^2-1}$ in the form $\frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomials in standard form. (TUSD)

Teaching Examples A-REI.2

Examples:

- Solve for x:

$$\begin{aligned} \circ \quad & \sqrt{x+2} = 5 \\ \circ \quad & \frac{7}{8} \sqrt{2x-5} = 21 \end{aligned}$$

MATHEMATICS COMMON CORE CURRICULUM UNIT #2 Algebra 2*
North Smithfield School Department

- $\frac{x+2}{x+3} = 2$
- $\sqrt[3]{3x-7} = -4$ (TUSD)

Teaching Examples A-REI.11

Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions. (Does not include logarithmic functions)

- Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.
- Given the following equations, determine the x value that results in an equal output for both functions.

$$f(x) = 3x - 2$$

$$g(x) = (x+3)^2 - 1$$

Graph the following system and give the solutions for $f(x) = g(x)$.

$$f(x) = |x+2|$$

$$g(x) = -\frac{1}{3}x + \frac{2}{3}$$

- Graph the following system and approximate the solutions for $f(x) = g(x)$.

$$f(x) = \frac{x+4}{2-x}$$

$$g(x) = x^3 - 6x^2 + 3x + 10$$
 (TUSD)

Teaching Examples F-IF.7

- In Algebra I, students looked at F-IF.7c as the relationship between zeros of quadratic functions and their factored forms.
- F-IF.7e links to F-TF.2 and 5 regarding the extension of trig functions.
- Logarithmic functions do not need to be addressed in Algebra II in terms of graphing.
- Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators, graphing programs, spreadsheets, or computer algebra systems to graph functions.

Examples:

- Graph the function $f(x) = \sqrt{x}$ by creating a table of values. Identify the key characteristics of the graph.
- Graph $f(x) = \sqrt[3]{x}$.

Teaching Examples F-IF.8

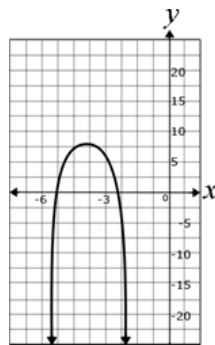
- In Algebra I, students focused on this standard with linear, exponential and quadratic functions.
- Example: Use rational and radical functions.

Teaching Examples F-IF.9

Example: Use rational and radical functions.

- Examine the functions below. Which function has the larger maximum? How do you know?

$$f(x) = -2x^2 - 8x + 20$$



(TUSD)

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North Smithfield School Department

Assessment Problems

A-REI.2

A-SSE.1

http://www.illustrativemathematics.org/illustrations/531	Delivery Trucks
http://www.illustrativemathematics.org/illustrations/1343	Delivery Trucks
http://www.illustrativemathematics.org/illustrations/89	Increasing or Decreasing?
http://www.illustrativemathematics.org/illustrations/215	Kitchen Floor Tiles
http://www.illustrativemathematics.org/illustrations/389	Mixing Candies
http://www.illustrativemathematics.org/illustrations/88	Mixing Fertilizer
http://www.illustrativemathematics.org/illustrations/187	Quadrupling Leads to Halving
http://www.illustrativemathematics.org/illustrations/1366	Radius of a Cylinder
http://www.illustrativemathematics.org/illustrations/390	A-SSE.A.1.b The Bank Account
http://www.illustrativemathematics.org/illustrations/23	The Physics Professor
http://www.illustrativemathematics.org/illustrations/90	Throwing Horseshoes

A-APR.6

http://www.illustrativemathematics.org/illustrations/825	Combined Fuel Efficiency
http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-6.html	Shmoop standard page
http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_6.pdf	Arithmetic with Polynomials - Worksheet 6
http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_6_ans.pdf	Arithmetic with Polynomials - Worksheet 6 Answer

Key

A-APR.7

http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-7.html
http://www.ixl.com/math/algebra-1/add-and-subtract-rational-expressions
http://www.ixl.com/math/algebra-1/multiply-and-divide-rational-expressions

A-REI.2

http://www.illustrativemathematics.org/illustrations/702	A-REI.A.2, A-CED.A.1	Basketball
http://www.illustrativemathematics.org/illustrations/391	Radical Equations	
http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-2.html	Shmoop standard page	
http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_2.pdf	Reasoning with Equation and Inequalities - Worksheet 2	
http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_2_ans.pdf	Reasoning with Equation and Inequalities -	

Worksheet 2 Answer Key

A-REI.11

http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-11.html	Shmoop REI.11 Quiz	
http://www.illustrativemathematics.org/illustrations/618	A-REI.B.4, A-REI.D.11	Two Squares are Equal
http://www.illustrativemathematics.org/illustrations/645	F-LE.2, F-LE.3, A-REI.11	Population and Food Supply

F.IF.7b

http://www.illustrativemathematics.org/illustrations/388	A-SSE.B.3, F-IF.C.7	Graphs of Quadratic Functions
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F.IF.7d

F.IF.8

http://www.illustrativemathematics.org/illustrations/640	F-IF.C.8.a	Which Function?
http://www.illustrativemathematics.org/illustrations/375	F-IF.C.8.a, A-REI.B.4.b	Springboard Dive
http://www.shmoop.com/common-core-standards/ccss-hs-f-if-8.html	Shmoop standard page	
http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_8.pdf	Functions Worksheet 5	
http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_8_ans.pdf	Functions Worksheet 5 – Answers	

F.IF.9

http://www.illustrativemathematics.org/illustrations/1279	F-IF.B.4, F-IF.C.9	Throwing Baseballs
http://www.shmoop.com/common-core-standards/ccss-hs-f-if-9.html	Shmoop standard page	
http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_9.pdf	Functions Worksheet 6	
http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_9_ans.pdf	Functions Worksheet 6 – Answers	

ACTIVITIES, PRODUCTS, PERFORMANCE, and ASSESSMENTS: see curriculum introduction

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|---------------------------------------|----------------------------|--|---------------------------------|
| 1. Application to real world problems | 6. Graphic organizers | 14. Problem/Performance based/common tasks | 18. Technology |
| 2. Creating charts/collecting data | 7. Graphing | 15. Real-life applications involving graphing | 19. Summarizing and note-taking |
| 3. Collaboration - interpersonal | 8. Interviews | 16. Represent numbers | 20. Tests and quizzes |
| 4. Conferencing | 9. Journals | 17. Rubrics/checklists (mathematical practice, modeling) | 21. Writing genres |
| 5. Exhibits | 10. KWL charts | | Arguments/ opinion Informative |
| • Warm ups | 11. Mathematical Practices | | |
| • Unit assessments | 12. Modeling ★ | | |
| • Semester/End of course exams | 13. Oral presentations | | |

MATHEMATICS COMMON CORE CURRICULUM UNIT #2 Algebra 2*

North Smithfield School Department

HIGHER ORDER THINKING SKILLS: Web's Depth of Knowledge 2 – 4 or Bloom's Taxonomy

Web's Depth of Knowledge

- skill/conceptual understanding
- strategic reasoning
- extended reasoning

Bloom's Taxonomy

- apply
- analyze
- synthesize/create
- evaluate

ADDITIONAL RESOURCES: see curriculum for specifics

Textbook

- *Algebra 2, McDougal Littell 2004*
- *Explorations, Holt McDougal*

Technology

- Computer lab
- Computer software that generate graphs of functions
- Computers
- Document camera
- Graphing calculator
- Graphing software
- Interactive boards
- LCD projectors
- Overhead graphing scientific

Websites

- <http://curriculum.northsmithfieldschools.com>
- <http://www.achieve.org/http://my.hrw.com>
- <http://www.illustrativemathematics.org/standards/practice>
- <http://www.ixl.com/standards/common-core/math/grade-8>
- <http://www.ixl.com/standards/common-core/math/high-school>
- <http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1>
- <http://www.ode.state.or.us/search/page/?id=3747>
- <http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S>
- <http://www.schools.utah.gov/CURR/mathsec/Core.aspx>
- <http://www.tusd1.org/contents/distinfo/curriculum/index.asp>
- www.commoncore.org/maps
- www.corestandards.org
- www.khanacademy.com
- www.ride.ri.gov

Materials

- Tables, graphs and equations of real-world applications that apply quadratic and exponential functions

VOCABULARY

- | | | | |
|-------------------------|------------------------|--------------------|-------------------------|
| • Asymptote | • Domain restriction | • Index | • Rational expression |
| • Closed set | • Linear factors | • Interval | • Root |
| • Coefficient | • End behavior | • Long division | • Solution |
| • Degree | • Expression | • Numerator | • Standard form |
| • Denominator | • Extraneous solution | • Polynomial | • Synthetic division |
| • Dependent variable | • Factor | • Radical | • Zero product property |
| • Difference of squares | • Factoring | • Radical equation | • Zeros |
| • Distributive property | • General form | • Radicand | |
| • Domain | • Independent variable | • Rational | |

MATHEMATICS COMMON CORE CURRICULUM UNIT #2 Algebra 2*
North Smithfield School Department

LESSON PLAN for UNIT _____

LESSONS

- Lesson # 1 Summary:

- Lesson #2 Summary:

- Lesson #3 Summary:

OBJECTIVES for LESSON # _____

- Materials/Resources:**

- Procedures:**
 - Lead -in

 - Step by step

 - Closure

- Instructional strategies:** see curriculum introduction

- Assessments:** see curriculum introduction
 - **Formative**

 - **Summative**