MATHEMATICS COMMON CORE CURRICULUM UNIT #2 Geometry*
North Smithfield School Department

TITLE OF UNIT: Similarity, Right Triangles, and Trigonometry **COURSE:** Geometry

DATE PRESENTED: 3/27/2013_____**DATE DUE:** _____ **LENGTH OF TIME:** Several weeks, quarter, semester

OVERVIEW OF UNIT:

In this unit we will gain an understanding of ratios and proportions and their properties. We will identify similar polygons and triangles. We will use similarity theorems to prove triangles are similar. We will investigate the properties of right triangles including the Pythagorean theorem and its converse. We will find side lengths of special right triangles as well as use trigonometry to solve them.

ESSENTIAL QUESTIONS

What makes polygons similar and how do you prove that they are?

How is trigonometry and the Pythagorean Theorem used to solve right triangles?

STANDARDS: Common Core Math Standards – Grade level Categories 9-12

Number and Quantity	Algebra	Functions	Modeling	Geometry	Statistics and Probability
☐ **The Real Number System N-RN**	☐ **Seeing Structure in Expressions A-SSE**	☐ **Interpreting Function F-If**	☐	☐ **Congruence G-CO**	☐ **Interpreting Categorical and Quantitative Data S-ID**
☐ **Quantities N-Q**	☐ **Arithmetic with Polynomials and Rational Expressions A-APR**	☐ **Building Functions F-BF**	☐	☐ **Similarity, Right Triangles, and Trigonometry G-SRT**	☐ **Making Inferences and Justifying Conclusions S-IC**
☐ **The Complex Number System N-CN**	☐ **Creating Equations A-CED**	☐ **Linear, Quadratic, and Exponential Models F-LE**		☐ **Circles G-c**	
☐ **Vector and Matrix Quantities N-VM**	☐ **Reasoning with Equations and Inequalities A-REI**	☐ **Trigonometric Functions F-TF**		☐ **Expressing Geometric Properties with Equations G-GPE**	
☐	☐	☐		☐ **Geometric Measurement and Dimensions G-GMD**	
☐	☐	☐		☐ **Modeling with Geometry G-MG**	

STANDARDS: Mathematical Practices grades K-12

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics ★
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

FOCUS MATHEMATICS STANDARDS: see curriculum _____ for specific standards, e.g.

- Understand similarity in terms of similarity transformations. G-SRT.1,2,3
- Prove theorems involving similarity. G-SRT.4,5
- Define trigonometric ratios and solve problems involving right triangles. G-SRT.6,7,8
- Apply geometric concepts in modeling situations. G-SRT.9
- Apply trigonometry to general triangles. G-SRT.10,11

Applied Learning Standards:

problem solving communication critical thinking research reflection/ evaluation

Expectations for Student Learning (High School only):

ENDURING UNDERSTANDING:

- When a line segment that does not pass through the center of a dilation is dilated, a parallel line segment is formed; line segments that pass through the center remain unchanged.
- The lengths of corresponding line segments in a figure and its dilation are proportional in the ratio given by the scale factor.
- Why do dilations involve scale factors?
- A dilation of a two-dimensional figure will create an image that is a similar figure to the original multiplied by a common scale factor/ratio.
- A similarity transformation is a combination of rigid motion and dilation.
- How can you describe the relationship between sides of similar figures?
- Triangles are similar if all corresponding angles are congruent and all corresponding sides are proportional.
- It can be shown using properties of similarity transformations that triangles are similar if two pairs of corresponding angles are congruent.

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

- What is the difference between similar and congruent figures?
- How can you show that two triangles are similar?
- The Pythagorean theorem can be proven using triangle similarity.
- Why does SSA not work to prove triangle congruence?
- Congruence and similarity can be used to determine missing angle measures and side lengths in geometric figures.
- How can you use congruence or similarity of pairs of figures to determine missing information about the figures?
- The ratios of the sides in any right triangle are properties of the angles in the triangle; these are called trigonometric ratios.
- The ratios of the sides in any right triangle are properties of the angles in the triangle; these are called trigonometric ratios.
- The sine and cosine of complementary angles are related.
- Trigonometric ratios and the Pythagorean theorem can be used to solve right triangles in applied problems.
- How can right triangles be used to solve real-world problems?
- Why do we need trigonometric ratios to solve some real-world problems involving right triangles?
- The area of any triangle can be written as a function of the lengths of two sides and the angle between them.(+)
- How can you derive the formula A = ½ ab sin(C) for the area of triangle?(+) The measures of the sides and angles of any triangle are related through the Law of Sines and the Law of Cosines.(+)
- How can you prove the Law of Sines? The Law of Cosines?(+)
- How can you use the Law of Sines or Law of Cosines to solve real-world problems?(+)
- The Law of Sines and the Law of Cosines can be used to solve all triangles in applied problems(+).
- How can you use the Law of Sines or Law of Cosines to solve real-world problems?(+)
- How do you determine unknown values in any triangle from given information?(+)

PRIOR KNOWLEDGE:

- Algebra 1
- Unit 1, Geometry

STUDENT OBJECTIVES, SKILLS and/or NEW KNOWLEDGE:

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles.
- Apply geometric concepts in modeling situations.
- Apply trigonometry to general triangles

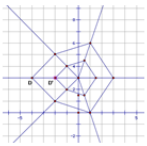
SUGGESTED PROBLEMS:

Teaching Examples

- A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Example:

- Draw a polygon. Pick a point and construct a dilation of the polygon with that point as the center. Identify the scale factor that you used. One solution:



Teaching Examples

- A similarity transformation is a rigid motion followed by a dilation.
- Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.
- Example:
- Are these two figures similar? Explain why or why not.

Teaching Examples

- Are all right triangles similar to one another? How do you know?
  TUSD
- http://www.illustrativemathematics.org/illustrations/602

Teaching Examples

Example:

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

- Prove that if two triangles are similar, then the ratio of corresponding altitudes is equal to the ratio of corresponding sides.
- To prove the Pythagorean theorem using triangle similarity:
- We can cut a right triangle into two parts by dropping a perpendicular onto the hypotenuse. Since these triangles and the original one have the same angles, all three are similar. Therefore

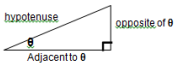
$$\frac{x}{a}=\frac{a}{c},\quad \frac{c-x}{b}=\frac{b}{c}$$
$$x=\frac{a^2}{c},\quad c-x=\frac{b^2}{c}$$
$$x+(c-x)=c$$
$$\frac{a^2}{c}+\frac{b^2}{c}=c$$
$$a^2+b^2=c^2$$

Teaching Examples

- http://www.illustrativemathematics.org/illustrations/1095
- http://www.illustrativemathematics.org/illustrations/651

Teaching Examples

- Students may use applets to explore the range of values of the trigonometric ratios as θ ranges from 0 to 90 degrees.



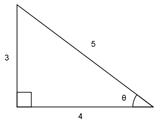
sine of $\theta = \sin\theta = \frac{opposite}{hypotenuse}$	cosecant of $\theta = \csc\theta = \frac{hypotenuse}{opposite}$
cosine of $\theta = \cos\theta = \frac{adjacent}{hypotenuse}$	secant of $\theta = \sec\theta = \frac{hypotenuse}{adjacent}$
tangent of $\theta = \tan\theta = \frac{opposite}{adjacent}$	cotangent of $\theta = \cot\theta = \frac{adjacent}{opposite}$

Essential knowledge and skills

- The ratios of the sides in any right triangle are properties of the angles in the triangle; these are called trigonometric ratios.
- The sine and cosine of complementary angles are related.

Teaching Examples

- What is the relationship between the sine and cosine of complementary angles?
  Example:
- Find the sine and cosine of angle θ in the triangle below. What do you notice?



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Academic vocabulary

- Complementary angles
- Cosine
- Sine

Mathematical Practices

- Construct viable arguments and critique the reasoning of others

Teaching Examples

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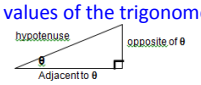

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- http://www.illustrativemathematics.org/illustrations/707
- http://www.illustrativemathematics.org/illustrations/1322
- http://www.illustrativemathematics.org/illustrations/1316
- http://www.illustrativemathematics.org/illustrations/607
- http://www.illustrativemathematics.org/illustrations/1345
- http://www.illustrativemathematics.org/illustrations/720
- http://www.illustrativemathematics.org/illustrations/710
- http://www.illustrativemathematics.org/illustrations/962

Teaching Examples(+)

- With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

Example:

- Find the area of the following triangle. Generalize by finding the area in terms of the side and angle labels instead of using the values.

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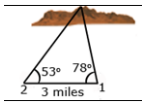
Teaching Examples (+)

Example:

- Tara wants to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78o. From the second position, the angle between the mountain and the first position is 53o. How can Tara determine the distance of the mountain from each position and what is the distance from each position?



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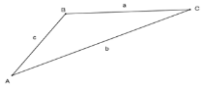
Teaching Examples(+)

- Find the area of the following triangle. Generalize by finding the area in terms of the side and angle labels instead of using the values.



Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$.

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$.

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ACTIVITIES, PRODUCTS, PERFORMANCE, and ASSESSMENTS: see curriculum introduction

1. Application to real world problems
2. Creating charts/collecting data
3. Collaboration - interpersonal
4. Conferencing
5. Exhibits
6. Graphic organizers
7. Graphing
8. Interviews
9. Journals
10. KWL charts
11. Mathematical Practices
12. Modeling ★
13. Oral presentations
14. Problem/Performance based/common tasks
15. Real-life applications involving graphing
16. Represent numbers
17. Rubrics/checklists (mathematical practice, modeling)
18. Technology
19. Summarizing and note-taking
20. Tests and quizzes
21. Writing genres
    Arguments/ opinion
    Informative

- Warm-ups
- Exit Slips

HIGHER ORDER THINKING SKILLS: Web's Depth of Knowledge 2 – 4 or Bloom's Taxonomy

Web's Depth of Knowledge

- skill/conceptual understanding
- strategic reasoning
- extended reasoning

Bloom's Taxonomy

- apply
- analyze
- synthesize/create
- evaluate

ADDITIONAL RESOURCES: see curriculum for specifics

Textbook

- *On Core Mathematics, Houghton Mifflin Harcourt*
- *McDougal Littell Geometry, Applying, Reasoning and Measuring*
- (Chapter 8, sections 8.1-8.7 and chapter 9, sections 9.1-9.7)

Technology

- Computers
- LCD projectors
- Interactive boards

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

Websites

- http://curriculum.northsmithfieldschools.com
- http://www.achieve.org/http://my.hrw.com
- http://www.illustrativemathematics.org/standards/practice
- http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1
- http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S
- http://www.tusd1.org/contents/distinfo/curriculum/index.asp
- www.commoncore.org/maps
- www.corestandards.org
- www.khanacademy.com
- www.ride.ri.gov

Materials

- Computer dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).
- Graph paper
- Instructional Resources/Tools
- Origami paper
- Protractor
- Pythagorean Puzzle - http://www.nsa.gov/academia/_files/collected_learning/high_school/geometry/pythagorean_puz Compass
- Reflection tool (e.g. Mira®).
- Ruler
- Scientific and/or Graphing calculators and other handheld technology such as TI-Nspire™.
- String
- Tracing paper (patty paper)
- Transparencies

VOCABULARY

- AA Similarity Theorem
- Acute angles
- Adjacent side
- Auxiliary line (+)
- Complementary angles
- Congruence
- Corresponding parts
- Cosine
- Dilation
- Geometric mean
- Hypotenuse
- Law of Cosines
- Law of Cosines (+)
- Law of Sines
- Law of Sines (+)
- Line segment
- Opposite side
- Parallel lines
- Proportion
- Proportionality
- Pythagorean Theorem
- Ratio
- Right triangle
- SAS Similarity Theorem
- Scale factor
- Scale factor
- Similarity
- Similarity
- Sine
- SSS Similarity Theorem
- Tangent
- Transformation
- Triangles
- Trigonometry

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

LESSON PLAN for UNIT _____

LESSONS

- **Lesson # 1 Summary:**

- **Lesson #2 Summary:**

- **Lesson #3 Summary:**

OBJECTIVES for LESSON # _____

- **Materials/Resources:**

- **Procedures:**
  - **Lead –in**
  - **Step by step**
  - **Closure**

- **Instructional strategies:** see curriculum introduction

- **Assessments:** see curriculum introduction
  - **Formative**
  - **Summative**

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