

MATHEMATICS COMMON CORE CURRICULUM UNIT #3 Algebra 2*

North Smithfield School Department

TITLE OF UNIT: Unit 3 Exponential and Logarithmic Functions **COURSE OR GRADE :** Algebra 2

DATE PRESENTED: _____ **DATE DUE:** _____ **LENGTH OF TIME:** Several weeks, quarter, semester

OVERVIEW OF UNIT:

Unit 3 standards will focus on exponential and logarithmic functions. Problems will focus on solving equations with these functions and graphing.

**ESSENTIAL QUESTION,
PROMPT, PROBLEM/UNIT**

STANDARDS: Common Core Math Standards – Grade level Categories 9-12

Number and Quantity	Algebra	Functions	Modeling	Geometry	Statistics and Probability
<input type="checkbox"/> The Real Number System N-RN	<input type="checkbox"/> Seeing Structure in Expressions A-SSE	<input type="checkbox"/> Interpreting Function F-If	<input type="checkbox"/>	<input type="checkbox"/> Congruence G-CO	<input type="checkbox"/> Interpreting Categorical and Quantitative Data S-ID
<input type="checkbox"/> Quantities N-Q	<input type="checkbox"/> Arithmetic with Polynomials and Rational Expressions A-APR	<input type="checkbox"/> Building Functions F-BF	<input type="checkbox"/>	<input type="checkbox"/> Similarity, Right Triangles, and Trigonometry G-SRT	<input type="checkbox"/> Making Inferences and Justifying Conclusions S-IC
<input type="checkbox"/> The Complex Number System N-CN	<input type="checkbox"/> Creating Equations A-CED	<input type="checkbox"/> Linear, Quadratic, and Exponential Models F-LE	<input type="checkbox"/>	<input type="checkbox"/> Circles G-c	
<input type="checkbox"/> Vector and Matrix Quantities N-VM	<input type="checkbox"/> Reasoning with Equations and Inequalities A-REI	<input type="checkbox"/> Trigonometric Functions F-TF	<input type="checkbox"/>	<input type="checkbox"/> Expressing Geometric Properties with Equations G-GPE	
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> Geometric Measurement and Dimensions G-GMD	
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> Modeling with Geometry G-MG	

STANDARDS: Mathematical Practices grades K-12

- | | | | | |
|---|--|--|---------------------------------------|--|
| 1. Make sense of problems and persevere in solving them | 3. Construct viable arguments and critique the reasoning of others | 5. Use appropriate tools strategically | 7. Look for and make use of structure | 8. Look for and express regularity in repeated reasoning |
| 2. Reason abstractly and quantitatively | 4. Model with mathematics ★ | 6. Attend to precision | | |

FOCUS MATHEMATICS STANDARDS:

- Understand solving equations as a process of reasoning and explain the reasoning. **A.REI.1**
- Write expressions in equivalent forms to solve problems. **A.SSE.3c, 4**
- Construct and compare exponential and logarithmic functions and solve problems. **F.LE.4**
- Represent and solve equations and inequalities graphically. **A.REI.4**
- Analyze functions using different representations. **F.IF.7e, 8b, 9**

Applied Learning Standards:

problem solving communication critical thinking research reflection/ evaluation

Expectations for Student Learning (High School only):

Problem Solving, Communication, Body of Knowledge, Responsibility

ENDURING UNDERSTANDING:

At the end of this unit, students will be proficient in the following:

- Solve equations involving exponential and logarithmic functions, including extraneous solutions.
- Graph exponential functions by hand, and more complex functions using technology.
- Rewrite exponential and logarithmic expressions.
- Represent and solve equations and inequalities graphically.
- Analyze functions using different representations.

PRIOR KNOWLEDGE:

Algebra 1

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STUDENT OBJECTIVES, SKILLS and/or NEW KNOWLEDGE:

- Define and use zero and negative exponents.
- Exponential growth and decay formulas
- Structure within expressions can be identified and used to factor or simplify the expression.
- A geometric series is the sum of terms in a geometric sequence.
- The sum of a finite geometric series with common ratio not equal to 1 can be written as a simple formula.
- Geometric series can be used to solve real-world problems.
- If $p(a) = 0$, then a is a zero of p .
- If a is a zero of p , then a is an x-intercept of the graph of $y = p(x)$.
- Solving a system of equations algebraically yields an exact solution; solving by graphing or by comparing tables of values yields an approximate solution.
- The x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.
- Key features of a graph or table may include intercepts; intervals in which the function is increasing, decreasing or constant; intervals in which the function is positive, negative or zero; symmetry; maxima; minima; end behavior; asymptotes; domain; range and periodicity.
- For a function of the form $f(t) = ab^t$, if $b > 1$ the function represents exponential growth; if $b < 1$ the function represents exponential decay.
- A function can be represented algebraically, graphically, numerically in tables, or by verbal descriptions.
- The solution to an exponential function can be found using logarithms.

SUGGESTED PROBLEMS:

Teaching Examples A-SSE.1

In Algebra I, students work with linear, exponential, and quadratic expressions. In Algebra II, students extend these concepts to general polynomials and rational expressions.

- Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.

Examples:

- What are the factors of $P(1+r)^n$? Which part(s) of this expression depend on P ?
 - a. A mixture contains A liters of liquid fertilizer in 10 liters of water. Write an expression for the concentration of fertilizer in the mixture, and explain what each part of the expression represents.
 - b. Another mixture contains twice as much fertilizer in the same amount of water as the mixture in part (a). Write an expression for the concentration of the new mixture, and explain why this concentration is not twice as much as the concentration of the first mixture. (TUSD)

Teaching Examples A-SSE.4

- In February, the Bezanson family starts saving for a trip to Australia in September. The Bezansons expect their vacation to cost \$5375. They start with \$525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip? (TUSD)

Teaching Examples A-REI.2

Examples:

- Solve for x :
 - $3e^x = 12$
 - $3 \cdot 2^x = 12$
 - $3 \log_5 x = 12$
 - $2 \ln(x - 2) = 18$

Teaching Examples A-REI.11

Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions. (Does not include logarithmic functions)

- Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.
- Given the following equations, determine the x value that results in an equal output for both functions.

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$$f(x) = e^{2x}$$

$$g(x) = 2 \ln 5x$$

Teaching Examples F.IF.7

- In Algebra I, students looked at F-IF.7c as the relationship between zeros of quadratic functions and their factored forms.
- F-IF.7e links to F-TF.2 and 5 regarding the extension of trig functions.
- Logarithmic functions do not need to be addressed in Algebra II in terms of graphing.
- Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators, graphing programs, spreadsheets, or computer algebra systems to graph functions.

Examples:

- Describe key characteristics of the graph of $f(x) = |x - 3| + 5$.
- Sketch the graph and identify the key characteristics of the function described below.

$$f(x) = 3 \cdot 2^x$$

$$g(x) = 2e^{5x}$$

Teaching Examples F.IF.8

- In Algebra I, students focused on this standard with linear, exponential and quadratic functions.

Example:

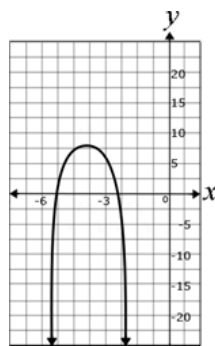
Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.

Teaching Examples F.IF.9

Example: Use exponential functions.

- Examine the functions below. Which function has the larger maximum? How do you know?

$$f(x) = -2x^2 - 8x + 20$$



(TUSD)

Assessment Problems

A-SSE.3

[Exponents: Negative exponents \(Algebra - V.3\)](#)

[Exponents: Multiplication with exponents \(Algebra - V.4\)](#)

[Exponents: Division with exponents \(Algebra - V.5\)](#)

[Exponents: Multiplication and division with exponents \(Algebra - V.6\)](#)

[Exponents: Power rule \(Algebra - V.7\)](#)

[Exponents: Simplify expressions involving exponents \(Algebra - V.8\)](#)

[Algebra review: Properties of exponents \(Geometry - A.3\)](#)

<http://www.schools.utah.gov/CURR/mathsec/Core/Secondary-II/II-3-A-SSE-3.aspx>

http://www.ode.state.or.us/wma/teachlearn/commoncore/mat.hs.sr.1.0asse.e.015_v1.pdf

A-SSE.4

<http://www.illustrativemathematics.org/illustrations/805>

Course of Antibiotics

<http://www.illustrativemathematics.org/illustrations/442>

Triangle Series

A-REI.2

<http://www.illustrativemathematics.org/illustrations/702> A-REI.A.2, A-CED.A.1

Basketball

<http://www.illustrativemathematics.org/illustrations/391>

Radical Equations

<http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-2.html>

Shmoop standard page

http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_2.pdf

Reasoning with Equation and Inequalities - Worksheet

2

http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_2_ans.pdf

Reasoning with Equation and Inequalities -

Worksheet 2 Answer Key

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A-REI.11

- <http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-11.html> Shmoop REI.11 Quiz
<http://www.illustrativemathematics.org/illustrations/618> A-REI.B.4, A-REI.D.11 Two Squares are Equal
<http://www.illustrativemathematics.org/illustrations/645> F-LE.2, F-LE.3, A-REI.11 Population and Food Supply

F.LE.4

- <http://www.illustrativemathematics.org/illustrations/370>
<http://www.illustrativemathematics.org/illustrations/570>
<http://www.illustrativemathematics.org/illustrations/369>
<http://www.illustrativemathematics.org/illustrations/760>
<http://www.illustrativemathematics.org/illustrations/214>
<http://www.illustrativemathematics.org/illustrations/382>
<http://www.illustrativemathematics.org/illustrations/638>
<http://www.illustrativemathematics.org/illustrations/784>
<http://www.shmoop.com/common-core-standards/ccss-hs-f-le-4.html> (new)

F.IF.7e

<http://www.illustrativemathematics.org/illustrations/803> (7e)

F.IF.9

- <http://www.illustrativemathematics.org/illustrations/1279> F-IF.B.4, F-IF.C.9 Throwing Baseballs
<http://www.shmoop.com/common-core-standards/ccss-hs-f-if-9.html> Shmoop standard page
http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_9.pdf Functions Worksheet 6
http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_9_ans.pdf Functions Worksheet 6 – Answers

ACTIVITIES, PRODUCTS, PERFORMANCE, and ASSESSMENTS: see curriculum introduction

- | | | | |
|---------------------------------------|----------------------------|--|---|
| 1. Application to real world problems | 6. Graphic organizers | 14. Problem/Performance based/common tasks | 18. Technology |
| 2. Creating charts/collecting data | 7. Graphing | 15. Real-life applications involving graphing | 19. Summarizing and note-taking |
| 3. Collaboration - interpersonal | 8. Interviews | 16. Represent numbers | 20. Tests and quizzes |
| 4. Conferencing | 9. Journals | 17. Rubrics/checklists (mathematical practice, modeling) | 21. Writing genres Arguments/ opinion Informative |
| 5. Exhibits | 10. KWL charts | | |
| | 11. Mathematical Practices | | |
| | 12. Modeling ★ | | |
| | 13. Oral presentations | | |
- Warm ups
 - Unit assessments
 - Semester/End of course exams

HIGHER ORDER THINKING SKILLS: Web's Depth of Knowledge 2 – 4 or Bloom's Taxonomy

Web's Depth of Knowledge

- skill/conceptual understanding
- strategic reasoning
- extended reasoning

Bloom's Taxonomy

- apply
- analyze
- synthesize/create
- evaluate

ADDITIONAL RESOURCES: see curriculum for specifics

Textbook

- *Algebra 2, McDougal Littell 2004*
- *Explorations, Holt McDougal*

Technology

- Computer lab
- Computer software that generate graphs of functions
- Computers
- Document camera
- Graphing calculator
- Graphing software
- Interactive boards
- LCD projectors
- Overhead graphing scientific

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Websites

- <http://curriculum.northsmithfieldschools.com>
- <http://www.achieve.org/http://my.hrw.com>
- <http://www.illustrativemathematics.org/standards/practice>
- <http://www.ixl.com/standards/common-core/math/grade-8>
- <http://www.ixl.com/standards/common-core/math/high-school>
- <http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1>
- <http://www.ode.state.or.us/search/page/?id=3747>
- <http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S>
- <http://www.schools.utah.gov/CURR/mathsec/Core.aspx>
- <http://www.tusd1.org/contents/distinfo/curriculum/index.asp>
- www.commoncore.org/maps
- www.corestandards.org
- www.khanacademy.com
- www.ride.ri.gov

Materials

- Tables, graphs and equations of real-world applications that apply quadratic and exponential functions

VOCABULARY

Academic vocabulary

- | | | |
|----------------------|-----------------------|--------------------|
| • Asymptote | • Expression | • Geometric series |
| • Domain | • Extraneous solution | • Range |
| • Exponential | • Finite series | • Root |
| • Exponential decay | • General form | • Standard form |
| • Exponential growth | • Geometric sequence | • Zeros |

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LESSON PLAN for UNIT _____

LESSONS

- Lesson #1** Summary:

- Lesson #2** Summary:

- Lesson #3** Summary:

OBJECTIVES for LESSON # _____

- Materials/Resources:**

- Procedures:**
 - **Lead -in**

 - **Step by step**

 - **Closure**

- Instructional strategies:** see curriculum introduction

- Assessments:** see curriculum introduction
 - **Formative**

 - **Summative**