

MATHEMATICS COMMON CORE CURRICULUM UNIT #5 Geometry*

North Smithfield School Department

TITLE OF UNIT: Circles

COURSE:: Geometry

DATE PRESENTED: _____ **DATE DUE:** _____ **LENGTH OF TIME:** Several weeks, quarter, semester

OVERVIEW OF UNIT:

In this unit, we will work with circles and explore their Connection to other geometric figures. We will investigate and prove what makes a circle similar to another circle. We will explore the relationships between the angles and arcs in a circle.

ESSENTIAL QUESTIONS

What is the relationship between central angles and inscribed angles in a circle?

How do you construct a circle that circumscribes a triangle?

How do you inscribe triangles and quadrilaterals in a circle?

What are the different types of segments relating to circles?

STANDARDS: Common Core Math Standards – Grade level Categories 9-12

Number and Quantity	Algebra	Functions	Modeling	Geometry	Statistics and Probability
<input type="checkbox"/> The Real Number System N-RN	<input type="checkbox"/> Seeing Structure in Expressions A-SSE	<input type="checkbox"/> Interpreting Function F-If	<input type="checkbox"/>	<input type="checkbox"/> Congruence G-CO	<input type="checkbox"/> Interpreting Categorical and Quantitative Data S-ID
<input type="checkbox"/> Quantities N-Q	<input type="checkbox"/> Arithmetic with Polynomials and Rational Expressions A-APR	<input type="checkbox"/> Building Functions F-BF	<input type="checkbox"/>	<input type="checkbox"/> Similarity, Right Triangles, and Trigonometry G-SRT	<input type="checkbox"/> Making Inferences and Justifying Conclusions S-IC
<input type="checkbox"/> The Complex Number System N-CN	<input type="checkbox"/> Creating Equations A-CED	<input type="checkbox"/> Linear, Quadratic, and Exponential Models F-LE	<input type="checkbox"/>	<input type="checkbox"/> Circles G-C	
<input type="checkbox"/> Vector and Matrix Quantities N-VM	<input type="checkbox"/> Reasoning with Equations and Inequalities A-REI	<input type="checkbox"/> Trigonometric Functions F-TF	<input type="checkbox"/>	<input type="checkbox"/> Expressing Geometric Properties with Equations G-GPE	
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> Geometric Measurement and Dimensions G-GMD	
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> Modeling with Geometry G-MG	

STANDARDS: Mathematical Practices grades K-12

- | | | | | |
|---|--|--|---------------------------------------|--|
| 1. Make sense of problems and persevere in solving them | 3. Construct viable arguments and critique the reasoning of others | 5. Use appropriate tools strategically | 7. Look for and make use of structure | 8. Look for and express regularity in repeated reasoning |
| 2. Reason abstractly and quantitatively | 4. Model with mathematics ★ | 6. Attend to precision | | |

FOCUS MATHEMATICS STANDARDS:

- Understand and apply theorems about circles. G-C.1,2,3,4
- Find arc lengths and areas of sectors of circles. G-C.5
- Translate between the geometric description and the equation for a conic section. G-GPE.2
- Use coordinates to prove simple geometric theorem algebraically. G-GPE.1
- Apply geometric concepts in modeling situations. ★ G-MG.1

Applied Learning Standards:

problem solving communication critical thinking research reflection/ evaluation

Expectations for Student Learning (High School only):

Problem Solving, Communication, Body of Knowledge, Proficiency in the Arts, Responsibility

ENDURING UNDERSTANDING:

At the end of this unit, students will be proficient in the following:

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles
- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorem algebraically.
- Apply geometric concepts in modeling situations. ★

PRIOR KNOWLEDGE:

- Algebra 1
- Units 1- 4

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STUDENT OBJECTIVES, SKILLS and/or NEW KNOWLEDGE:

- It can be proven that all circles are similar.
- Central, inscribed, and circumscribed angles on a circle are related to each other.
- The length of a chord on a circle is related to the inscribed, circumscribed, and central angles defined by the endpoints of the chord.
- The radius of a circle is perpendicular to the tangent at the point of intersection.
- Every triangle has a unique inscribed circle and a unique circumscribed circle
- Quadrilaterals inscribed in a circle have properties that can be stated as theorems and proven.
- How can you prove that the sum of opposite angles in an inscribed quadrilateral equals 180° ?
- From any point outside a given circle, two tangent lines to the circle can be constructed.(+)
- Why does a point have to be outside a circle in order to be able to construct a tangent line from that point to the circle?(+)
- Arc lengths on a circle are proportional to the radius; this fact follows from the similarity of circles.
- The ratio of an arc length to the radius defines a unit of measurement for the central angle that intercepts that arc; this unit is called the radian.
- The radian measure of an angle is the constant of proportionality.
- The formula for the area of a sector can be derived by exploring the relationship between the central angle and the area formula for a circle.
- How is an arc length on a circle related to the radius? How can you derive this relationship?
- How do you convert an angle from degree measure to radian measure? Why does this work?
- How do you find the arc length of a circle in degrees? Radians?
- How can you derive the formula for the area of a sector?
- The equation of a parabola can be derived given its focus and directrix.
- What information do you need to derive the equation of a parabola and why?
- Coordinate geometry can be used to solve simple geometric theorems algebraically.
- Geometric shapes and their properties can be used to model real-world objects.
- How can you model objects in your classroom as geometric shapes?

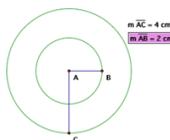
SUGGESTED PROBLEMS:

Teaching Examples

- How can you prove that all circles are similar?

Example:

- Draw or find examples of several different circles. In what ways are they related? How can you describe this relationship in terms of geometric ideas? Form a hypothesis and prove it.



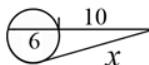
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Teaching Examples

- Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.



- Find the unknown length in the picture below.



- Solution:

The theorem for a secant segment and a tangent segment that share an endpoint not on the circle states that for the picture below secant segment QR and tangent segment SR share an endpoint, R, not on the circle. Then the length of SR squared is equal to the product of the lengths of QR and KR.



$$x^2 = 16 \cdot 10$$

So for the example above: $x^2 = 160$

$$x = \sqrt{160} = 4\sqrt{10} \approx 12.6$$

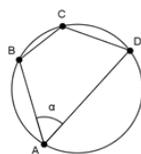
- How does the angle between a tangent to a circle and the line connecting the point of tangency and the center of the circle change as you move the tangent point?

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Teaching Examples

- Given the inscribed quadrilateral below prove that $\angle B$ is supplementary to $\angle D$.



Teaching Examples (+)

- Students may use geometric simulation software to make geometric constructions (+)

Example: (+)

To construct a tangent line to circle C:

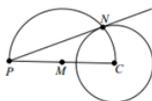
Draw a point P outside of the circle.

Connect center of the circle C with point P and construct the perpendicular bisector of the segment. Label the point where the perpendicular bisector and the segment meet M.

With M as the center draw a half circle through P and C.

Construct a point at the intersection of the half circle and circle C, label this point N.

Draw a line through P and N, Line PN is the tangent to circle C.



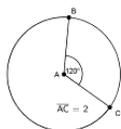
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Teaching Examples

- Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angles, arc lengths are proportional to the radius. Use this as a basis for introducing the radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.
- Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector.

Example:

- Find the area of the sectors below. What general formula can you develop based on this information?



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ACTIVITIES, PRODUCTS, PERFORMANCE, and ASSESSMENTS: see curriculum introduction

- | | | | |
|---------------------------------------|----------------------------|--|---|
| 1. Application to real world problems | 6. Graphic organizers | 14. Problem/Performance based/common tasks | 18. Technology |
| 2. Creating charts/collecting data | 7. Graphing | 15. Real-life applications involving graphing | 19. Summarizing and note-taking |
| 3. Collaboration - interpersonal | 8. Interviews | 16. Represent numbers | 20. Tests and quizzes |
| 4. Conferencing | 9. Journals | 17. Rubrics/checklists (mathematical practice, modeling) | 21. Writing genres Arguments/ opinion Informative |
| 5. Exhibits | 10. KWL charts | | |
| | 11. Mathematical Practices | | |
| | 12. Modeling ★ | | |
| | 13. Oral presentations | | |
| • Warm ups | | | |
| • Exit Slips | | | |
| • Polls | | | |

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HIGHER ORDER THINKING SKILLS: Web's Depth of Knowledge 2 – 4 or Bloom's Taxonomy

Web's Depth of Knowledge

- skill/conceptual understanding
- strategic reasoning
- extended reasoning

Bloom's Taxonomy

- apply
- analyze
- synthesize/create
- evaluate

ADDITIONAL RESOURCES: see curriculum for specifics

Textbook

- *McDougal Littell Geometry, Applying, Reasoning and Measuring 10.1-10.5 11.4 11.5*
- *Houghton Mifflin Harcourt, On Core Mathematics Geometry Unit 7*

Technology

- Computers
- LCD projectors
- Interactive boards

Websites

- <http://curriculum.northsmithfieldschools.com>
- <http://www.achieve.org/http://my.hrw.com>
- <http://www.illustrativemathematics.org/standards/practice>
- <http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1>
- <http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S>
- <http://www.tusd1.org/contents/distinfo/curriculum/index.asp>
- www.commoncore.org/maps
- www.corestandards.org
- www.khanacademy.com
- www.ride.ri.gov

Materials

- Compass
- Computer dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).
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- Graph paper
- Instructional Resources/Tools
- Origami paper
- Protractor
- Protractor
- Pythagorean Puzzle - http://www.nsa.gov/academia/files/collected_learning/high_school/geometry/pythagorean_puz Compass
- Reflection tool (e.g. Mira®).
- Ruler
- Ruler
- Scientific and/or Graphing calculators and other handheld technology such as TI-Nspire™.
- String
- Tracing paper (patty paper)
- Transparencies

VOCABULARY

- Arc length
- Central angle
- Central angle
- Chord
- Circle
- Circumscribed
- Diameter
- Inscribed angle
- Intercepted arc
- Point of tangency
- Radian measure
- Radii
- Radius
- Sector
- Tangent
- Tangent line (+)

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LESSON PLAN for UNIT _____

LESSONS

- Lesson # 1 Summary:

- Lesson #2 Summary:

- Lesson #3 Summary:

OBJECTIVES for LESSON # _____

- Materials/Resources:**

- Procedures:**
 - Lead -in

 - Step by step

 - Closure

- Instructional strategies:** see curriculum introduction

- Assessments:** see curriculum introduction
 - **Formative**

 - **Summative**