MATHEMATICS COMMON CORE CURRICULUM UNIT #6 Algebra 2*
North Smithfield School Department

TITLE OF UNIT: Unit 6 Inferences and Conclusions from Data **COURSE:** Algebra 2

DATE PRESENTED: ______________ **DATE DUE:** __________________ **LENGTH OF TIME:** Several weeks, quarter, semester

OVERVIEW OF UNIT:

Unit 6 standards will summarize, represent, and interpret data on a single count or measurement variable focusing on using the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. This unit will also have students make inferences and justify conclusions from sample surveys, experiments, and observational studies. Finally, students will use probability to evaluate outcomes of decisions.

ESSENTIAL QUESTION, PROMPT, PROBLEM/UNIT

STANDARDS: Common Core Math Standards – Grade level Categories 9-12

Number and Quantity	Algebra	Functions	Modeling	Geometry	Statistics and Probability
☐ The Real Number System N-RN	☐ Seeing Structure in Expressions A-SSE	☐ Interpreting Function F-If	☐	☐ Congruence G-CO	☐ Interpreting Categorical and Quantitative Data S-ID
☐ Quantities N-Q	☐ Arithmetic with Polynomials and Rational Expressions A-APR	☐ Building Functions F-BF	☐	☐ Similarity, Right Triangles, and Trigonometry G-SRT	☐ Making Inferences and Justifying Conclusions S-IC ☐ Using Probability to Make Decisions
☐ The Complex Number System N-CN	☐ Creating Equations A-CED	☐ Linear, Quadratic, and Exponential Models F-LE		☐ Circles G-c	
☐ Vector and Matrix Quantities N-VM	☐ Reasoning with Equations and Inequalities A-REI	☐ Trigonometric Functions F-TF		☐ Expressing Geometric Properties with Equations G-GPE	
☐	☐	☐		☐ Geometric Measurement and Dimensions G-GMD	
☐	☐	☐		☐ Modeling with Geometry G-MG	

STANDARDS: Mathematical Practices grades K-12

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics ★
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

FOCUS MATHEMATICS STANDARDS:

- Summarize, represent, and interpret data on single count or measurement variable. S.ID.4
- Understand and evaluate random processes underlying statistical experiments. S.IC.1,2
- Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3,4,5,6
- Use probability to evaluate outcomes of decisions. S.MD.(+)6,7

Applied Learning Standards:

problem solving communication critical thinking research reflection/ evaluation

Expectations for Student Learning (High School only):

Problem Solving, Communication, Body of Knowledge, Responsibility

ENDURING UNDERSTANDING:

At the end of this unit, students will be proficient in the following:

- Understand how to find percents of data and probabilities of events associated with normal distributions.
- Understand the different methods for gathering data about a population.
- Understand how randomization relates to sample surveys, experiments, and observational studies.
- Understand the difference between a control group and a treated group.
- Understand how to use data from sample surveys, observational studies, and experiments to draw inferences and justify conclusions.
- Determine the significance of experimental results.
- Evaluate reports based on data.
- Understand you to determine if a game is fair.
- Analyze decisions and strategies using probability concepts.

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

PRIOR KNOWLEDGE:
Algebra 1 and Geometry

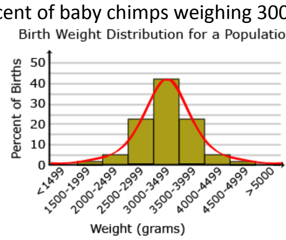
STUDENT OBJECTIVES, SKILLS and/or NEW KNOWLEDGE:

- A normal distribution can describe some, but not all, data sets.
- Each normal distribution has a well-defined mean and standard deviation.
- The mean and standard deviation of a data set can be used to find the best-fit normal distribution for that data set.
- The normal distribution of a set of population data can be used to estimate population percentages.
- If a model is appropriate for a given situation, the experimental probability of an event will approach the theoretical probability as the sample size increases.
- Experiments must be repeated to verify a model.
- Large numbers of trials can be performed using computer simulations.
- Sample surveys, experiments and observational studies are three ways to collect data.
- In an observational study, assignment of subjects into a treated group versus a control group is outside the control of the investigator.
- In an observational study, the randomization is inherent in the population.
- In controlled experiments, each subject is randomly assigned to a treated group or a control group before the start of the treatment.
- A sample survey allows you to collect data from a subset of the population, and draw inferences about the larger population.
- In a sample survey it is important to collect data from a random sampling that mimics the larger population.
- Data from a sample survey can be used to estimate a population mean or proportion and then develop a margin of error from a simulation model.
- Simulations of random samplings and experiments can be used to support inferences from the data.
- Data from a randomized experiment can be used to compare two treatments.
- Reported data may be misleading due to, for example, sample size, biased survey sample, choice of interval scale, unlabeled scale, uneven scale, and outliers.
- Probabilities can be used to make fair decisions. (+)
- Probabilities can be used to analyze and evaluate decisions and strategies. (+)

SUGGESTED PROBLEMS:

Teaching Examples S.ID.4

Examples:
- Determine which situation(s) is/are best modeled by a normal distribution. Explain your reasoning.
  - Annual income of a household in the U.S.
  - Weight of babies born in one year in the U.S.
- The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000-3999 grams.



(TUSD)

Teaching Examples S.IC.1

Example:
- Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.

  1. Describe the parameter of interest and a statistic the students could use to estimate the parameter.
  2. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
  3. The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness.

4. Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above. (TUSD)

From: illustrativemathematics.org

Teaching Examples S.IC.2

For S-IC.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.

- Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using computer random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials.
- The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model-building verification process.

Examples:

- Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability?

- A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (TUSD)

Teaching Examples S.IC.3

In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

- Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment.

- In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment. (TUSD)

Teaching Examples S.IC.4

- For S-IC.4 and 5, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
- Students may use computer-generated simulation models based upon the results of sample surveys to estimate population statistics and margins of error. (TUSD)

Teaching Examples S.IC.5

- Treatment is a term used in the context of an experimental design to refer to any prescribed combination of values of explanatory variables. For example, one wants to determine the effectiveness of weed killer. Two equal parcels of land in a neighborhood are treated, one with a placebo and one with weed killer, to determine whether there is a significant difference in effectiveness in eliminating weeds. (TUSD)

Teaching Examples S.IC.6

- Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion.
- As a strategy, collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.

Example:

- A reporter used the two data sets below to calculate the mean housing price in Arizona as $629,000. Why is this calculation not representative of the typical housing price in Arizona?

  - King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}
  - Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000} (TUSD)

Teaching Examples (+)S.MD.6

Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.

A game is fair if all players have an equal chance of winning. For more complicated games, it is often useful to calculate the expected value of the game (i.e., average winnings) for each player. Students begin to work with expected values in middle school.

Examples:

- John has designed a game using 2 dice. The rules state that Player A will get ten points if after rolling the dice the product is prime. Player B will get one point if the product is not prime. John feels this scoring system is reasonable because there are many more ways to get a non-prime product.

  Is John's game fair? Explain why or why not.

- Suppose that a blood test indicates the presence of a particular disease 97% of the time when the disease is actually present. The same test gives false positive results 0.25% of the time. Suppose that one percent of the population actually has the disease. Suppose your blood test is positive. How likely is it that you actually have the disease? (TUSD)

Teaching Examples (+)S.MD.7
Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.

A game is fair if all players have an equal chance of winning. For more complicated games, it is often useful to calculate the expected value of the game (i.e., average winnings) for each player. Students begin to work with expected values in middle school.

Examples:
- (The Monty Hall problem) Suppose you're on Let's Make a Deal, and you're playing the big deal of the day: you are given the choice of three curtains. Behind one curtain is a new car; behind the other two are zonks. You pick curtain number 1. The host, who knows where the car is, opens curtain number 3, which has a zonk. The host then says, "Do you want to switch curtains?" Is it better to switch or to keep your first choice, and why?
- Wanda, the Channel 1 weather person, said there was a 30% chance of rain on Saturday and a 30% chance of rain on Sunday. It rained both days, and Wanda's station manager is wondering if she should fire Wanda.

  - Suppose Wanda's calculations were correct and there was a 30% chance of rain each day. What was the probability that there would be rain on both days?
  - Do you think Wanda should be fired? Why or why not?
  - Wanda is working on her predictions for the next few days. She calculates that there is a 20% chance of rain on Monday and a 20% chance of rain on Tuesday. If she is correct, what is the probability that it will rain on at least one of these days?

- From: Connected Mathematics, "What Do You Expect?" (TUSD)

Assessment Problems

S-ID.4 Basic
- http://www.illustrativemathematics.org/illustrations/216
- http://www.illustrativemathematics.org/illustrations/1020
- http://www.shmoop.com/common-core-standards/ccss-hs-s-id-4.html

S-ID.4 Advanced
- http://www.illustrativemathematics.org/illustrations/1218

S-IC.1 Basic
- http://www.illustrativemathematics.org/illustrations/186
- http://www.illustrativemathematics.org/illustrations/122
- http://www.illustrativemathematics.org/illustrations/191
- http://www.illustrativemathematics.org/illustrations/123
- http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-1.html

S-IC.2 Basic
- http://www.illustrativemathematics.org/illustrations/125
- http://www.illustrativemathematics.org/illustrations/244
- http://www.illustrativemathematics.org/illustrations/1099
- http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-2.html

S-IC.3 Basic
- http://www.illustrativemathematics.org/illustrations/1029
- http://www.illustrativemathematics.org/illustrations/1100
- http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-3.html

S-IC.4 Basic
- http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-4.html

S-IC.5 Basic
- http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-5.html

S-IC.6 Basic
- http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-6.html

S-MD.6 Basic
- http://www.shmoop.com/common-core-standards/ccss-hs-s-md-6.html

S-MD.7 Basic
- http://www.illustrativemathematics.org/illustrations/1197
- http://www.shmoop.com/common-core-standards/ccss-hs-s-md-7.html

ACTIVITIES, PRODUCTS, PERFORMANCE, and ASSESSMENTS: see curriculum introduction

1. Application to real world problems
2. Creating charts/collecting data
3. Collaboration - interpersonal
4. Conferencing
5. Exhibits
6. Graphic organizers
7. Graphing
8. Interviews
9. Journals
10. KWL charts
11. Mathematical Practices
12. Modeling ★
13. Oral presentations
14. Problem/Performance based/common tasks
15. Real-life applications involving graphing
16. Represent numbers
17. Rubrics/checklists (mathematical practice, modeling)
18. Technology
19. Summarizing and note-taking
20. Tests and quizzes
21. Writing genres
    Arguments/ opinion
    Informative

- Warm ups
- Unit assessments
- Semester/End of course exams

HIGHER ORDER THINKING SKILLS: Web's Depth of Knowledge 2 – 4 or Bloom's Taxonomy

Web's Depth of Knowledge
- skill/conceptual understanding
- strategic reasoning
- extended reasoning

Bloom's Taxonomy
- apply
- analyze
- synthesize/create
- evaluate

ADDITIONAL RESOURCES: see curriculum for specifics

Textbook
- *Algebra 2, McDougal Littell 2004*
- *Explorations, Holt McDougal*

Technology
- Computer lab
- Computer software that generate graphs of functions
- Computers
- Document camera
- Graphing calculator
- Graphing software
- Interactive boards
- LCD projectors
- Overhead graphing scientific

Websites
- http://curriculum.northsmithfieldschools.com
- http://www.achieve.org/http://my.hrw.com
- http://www.illustrativemathematics.org/standards/practice
- http://www.ixl.com/standards/common-core/math/grade-8
- http://www.ixl.com/standards/common-core/math/high-school
- http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1

*Referenced templates from *Common Core Curriculum Maps, English Language Arts* and *The Understanding By Design Guide to Creating High Quality Units*

- http://www.ode.state.or.us/search/page/?id=3747
- http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S
- http://www.schools.utah.gov/CURR/mathsec/Core.aspx
- http://www.tusd1.org/contents/distinfo/curriculum/index.asp
- www.commoncore.org/maps
- www.corestandards.org
- www.khanacademy.com
- www.ride.ri.gov

Materials

- Tables, graphs and equations of real-world applications that apply quadratic and exponential functions

VOCABULARY

Academic vocabulary

- Bell curve
- Mean
- Median
- Bell curve
- Mode
- Normal distribution
- Mean
- Sigma
- Median
- Standard deviation
- Mode
- Bell curve
- Variance
- Normal distribution
- Mean
- Sigma
- Median
- Standard deviation
- Mode
- Normal distribution
- Variance
- Sigma
- Standard deviation
- Variance

Academic vocabulary

- Control group
- Line of best fit
- Observational study
- Outliers
- Random sample
- Randomization
- Regression
- Sample size
- Survey

Academic vocabulary

- Expected value
- Fair games
-
- False negative
- False positive
- Least-squares regression
- Random number generator

LESSON PLAN for UNIT ________________

LESSONS

- **<u>Lesson # 1</u> Summary:**
- **<u>Lesson #2</u> Summary:**
- **<u>Lesson #3</u> Summary:**

OBJECTIVES for LESSON # ________________

- **Materials/Resources:**
- **Procedures:**
  - **Lead –in**
  - **Step by step**
  - **Closure**
- **Instructional strategies:** see curriculum introduction
- **Assessments:** see curriculum introduction
  - **Formative**
  - **Summative**

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